Paper number: 8(2010)3,184, 181-188

DESIGN OF CONSTRUCTIONS WITH RESPECTS TO FATIGUE AND FRACTURE MECHANICS

Dr Stevan Maksimović* *
Military Technical Institute, Belgrade
Marija Blažić
Military Technical Institute, Belgrade
Mirko Maksimović
Belgrade waterworks and sewerage

An attention in this paper is focused on developing computation procedures during design of flight structures with respects fatigue and fracture mechanics Here are developed analytic expressions for the stress intensity factor (SIF) in 3-D solid type structural components for crack growth analyses. Damages of structural components are modeled with semi-elliptic surface cracks. Structural components are under cyclic loads and load spectra. Results of presented analytic model of FIN are compared with finite element results. Good agreement between presented analytic and finite element results is obtained. Strength analysis with respect static fracture mechanics is illustrated on nose landing gear problem of light training aircraft.

Key words: Fatigue of structure, Fracture mechanics, Crack in 3-D solids, Analytic model, Life estimation, Metal structures.

INTRODUCTION

Structural integrity analysis of aging aircraft is a critical necessity in view of the increasing numbers of such aircraft in general aviation, the airlains and the military. Damage tolerance analysis can be used to assess the remaining life of aircraft in service. In a damage tolerant design/analysis initial cracks are considered to be present at each of the critical components in the structure and these initial cracks are allowed to grow. At each stage in the life of the structure the current crack lengths are evaluated and the associated stress intensity factors K are computed. These values of K are then used to evaluate both the residual strength and the crack growth rate associated with each crack. In aerospace industry the initial cracks tend to be quite small. Thus at each stage of the life of structural components it is necessary to analyse of a complex two-dimensional (2-D) or three-dimensional (3-D) crack under arbitrary loading. Due to the singularity along crack front this requires to use singular finite elements.

Design approach of structural components with initial proposed cracks at the potential critical area of complex construction is known as damage tolerance approach [1]. In practical design numerical simulations based on finite element method (FEM) are efficient method do determine critical locations at structural components with respect the fatigue and fracture mechanics. Residual life estimations of damaged structural components under cyclic loading requires determination of various fracture mechanics parameters [2,3,6-9]. Key parameter in fracture mechanics analysis is stress intensity parameter K.

In a damage tolerance design of thin-walled structures the initial cracks are proposed through the thickness of structural components. However in situations of robust structural components and constructions initial cracks must be defined as surface cracks. In aircraft constructions, typical “3-D” solid with initial surface damages are structures of landing gears and wing/fuselage joints.

In contrast to the thin-walled construction considerable complex problem represent defining SIF in the analytic form for 3-D structure. There exist a lot of attemption to derive analytic expressions of SIF to semi-elliptic surface crack [10,12,15]. In this work one analytic model for determination

* *Military Technical Institute, Ratka Resanovića 1, Belgrade; s.maksimovic@nadlanu.com*

SIF for “3-D” semi-elliptic surface crack is proposed. For determination of “3-D” semi-elliptic surface crack can be used finite element method (FEM). For that purpose are used special 3-D singular finite elements. Singular finite elements are very accurate method for SIF computation, but crack growth analysis is very complex. On because of that reason in this work an attention is focused to deriving quality analytic expressions for determination of stress intensity factors and crack growth models.

BASIC RELATIONS TO CRACK GROWTH ANALYSIS

In the residual life estimation of structural components with respects fatigue and fracture mechanics can be used various crack growth models. For this purpose for the fatigue crack growth programs are the incorporation of the ability to read aircraft spectra of unlimited size, generation of common aircraft fatigue load blocks and the incorporation of crack-growth models which include load-interaction effects such as retardation due to overloads and acceleration due to underloads. In this investigation Forman and Mettu [4] crack-growth model is used. This crack-growth model is incorporated im many commercial fatigue life softwate codes. This crack growth model is defined in the next form:

$$\frac{da}{dN} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^n \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{max}}{K_c}\right)^q} \quad (1)$$

where ***N*** is the number of applied fatique cycles

a - crack lenght

R (= $\sigma_{min}/\sigma_{max}$) - is the stress ratio

$\Delta K(= K_{max} - K_{min})$ - is the stress intensity factor (SIF) range

C, n, p, q - are empirically determined constants under cyclic loads,

f - is the crack opening function

ΔK_{th} - is the threshold stress intensity factor

and K_c is the critical stress intensity factor. This equation provides a direct formulation of the stress-ratio effect. Newman is defined the crack opening function in form:

$$f = \frac{K_{op}}{K_{max}} = \begin{cases} \max\left(R, A_0 + A_1 R + A_2 R^2 + A_3 R^3\right); R \geq 0 \\ A_0 + A_1 R ; \quad -2 \leq R < 0 \end{cases} \quad (2)$$

where previous coefficient are defined as:

$$A_0 = \left(0.825 - 0.34\alpha + 0.05\alpha^2\right)\left[\cos\left(\frac{\pi}{2}\frac{S_{max}}{\sigma_0}\right)\right]^{1/\alpha} \quad (3)$$

$$A_1 = \left(0.415 - 0.071\alpha\right)\frac{S_{max}}{\sigma_0} \quad (4)$$

$$A_2 = 1 - A_0 - A_1 - A_3 \quad (5)$$

$$A_3 = 2A_0 + A_1 - 1 \quad (6)$$

In these equations

α is plane stress/strain constraint factor and

σ_{min}/σ_0 is the ratio of the maximum applied stress to the flow stress.

DETERMINATION OF STRESS INTENSITY FACTORS FOR SEMI-ELLIPTIC SURFACE CRACKS

For crack growth analysis, in accordance of eq. (1), it is necessary to be known stress intensity factor (SIF), K, in the analytic form. Here is considered problem threedimensional structural elements with semi-elliptic surface crack, Fig.1.

For determination of SIF at critical locations, points A and B in Fig. 1, here ‘’layered” model [5,14] is used.

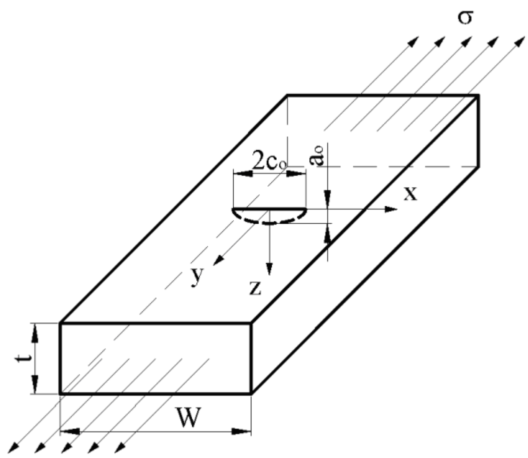

Figure 1. part one - Model 3-D solid with surface semi-elliptic crack

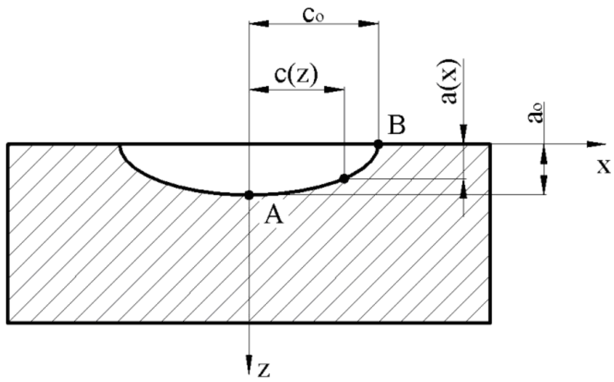

Figure 1. part two - Model 3-D solid with surface semi-elliptic crack

This model divide solid to layers in horizontal and vertical planes making two systems in which: layers in plane *x -y* represent conventional plates with central cracks and layers in plane *y-z* represent plates with edge cracks.

This systems are conjugated across distribution of pressure ***p****, acting to surfaces of crack of each system and causing that these displacements of two systems need to be equal. Using pressure distribution to surface of crack, the stress intensity factors at the points A and B may be determined as:

$$K_A = \int_0^{a_0} p^*(0,z)\, g(z,a_0)_{c,p}\, dz \tag{7}$$

$$K_B = \int_{-C_0}^{C_0} \left[\sigma - p^*(x,0)\right] g(x,c_0)_{i,p}\, dx \tag{8}$$

Function $g(x,\xi)$ that is used in previous

relations is general influence of function for central crack, g_{ck}, or edge crack, g_{ip} . The influence function is defined as:

$$g(x,\xi) = \frac{E}{K(\xi)} \frac{\partial v(x,\xi)}{\partial \xi} \tag{9}$$

where K(ξ) is the stress intensity factor for some load distributions which act to crack length ξ, and v (x,ξ) is the displacement of crack surface at any point ***x***. Without the complete theoretical consideration here are given final expressions for stress intensity factors for semi-elliptic surface crack at points A and B in accordance to Fig.1,

$$K_A = \sigma \sqrt{\pi a} \sum_{i=0}^{3}\sum_{j=0}^{3} A_{ij} \left(\frac{c}{a}\right)^{i/2} \left(\frac{a}{t}\right)^{i} \tag{10}$$

$$K_B = \sigma \sqrt{\pi a} \sum_{i=0}^{3}\sum_{j=0}^{3} B_{ij} \left(\frac{c}{a}\right)^{i/2} \left(\frac{a}{t}\right)^{i} \tag{11}$$

where and are the crack lengths and thickness are defined in Fig. 1, until ***Aij*** and ***Bij*** represent corresponding the influence functions.

NUMERICAL SIMULATIONS

To illustrate numerical simulations to residual life estimations and strength analysis with fracture mechanics here are numerical examples are included.

Example 1: Crack growth analysis

Analytic expressions for SIF's of semi-elliptic crack to surface of 3-D solid, defined in previous consideration and crack growth model, are incorporated in software package "LOM-3"[11].

To illustrate numerical simulation of crack growth in 3-D solids here is included problem of thick plate with surface crack under tension cyclic loading. Here is established analytic computation procedure that is illustrated for crack growth analysis of 3-D solid under load spectra. Crack growth analysis under load spectra is defined in Table 1.

Table 1. Load spectra

Ni	Smin [MPa]	Smax [MPa]
6000	0	500
6000	100	400

Crack growth analysis made to the next material and geometric properties of panel with semi-elliptic surface crack, as shown in Fig.1.

Width of crack	a = .100E-02 [m]
Height of crack	c = .100E-02 [m]
Thickness	t = .200E-01 [m]
Width	W = .200E-01 [m]
Parisso's coef.	C_p = .300E-10
Paris exsponent	n_p = .250E+01
Fracture toughness	K_{ic} = .500E+02

Table 2. Širenje 3-D prskotine

a [m]	N (F15A)	N (F15B)	N (german)	a [m]
0.001	0.00E+00	0.001	0.00E+00	0.001
0.0013	4.95E+03	0.0013	3.91E+03	0.0013
0.0014	6.00E+03	0.0015	6.00E+03	0.0015

0.0015	1.20E+04	0.0017	1.20E+04	0.0017		0.0034	3.00E+04	0.0036	2.60E+04	0.0037
0.0018	1.51E+04	0.0019	1.42E+04	0.002		0.0037	3.42E+04	0.0038	2.69E+04	0.004
0.0021	1.77E+04	0.0022	1.60E+04	0.0023		0.0038	3.60E+04	0.0041	2.77E+04	0.0042
0.0021	1.80E+04	0.0025	1.75E+04	0.0025		0.0041	3.70E+04	0.0044	2.84E+04	0.0045
0.0023	2.40E+04	0.0026	1.80E+04	0.0027		0.0044	3.79E+04	0.0047	2.90E+04	0.0048
0.0026	2.59E+04	0.0029	2.25E+04	0.003		0.0047	3.88E+04	0.005	2.96E+04	0.0051
0.0029	2.75E+04	0.003	2.40E+04	0.0031		0.0049	3.96E+04	0.0051	3.00E+04	0.0054
0.0032	2.89E+04	0.0033	2.51E+04	0.0034		0.0052	4.03E+04	0.0054	3.19E+04	

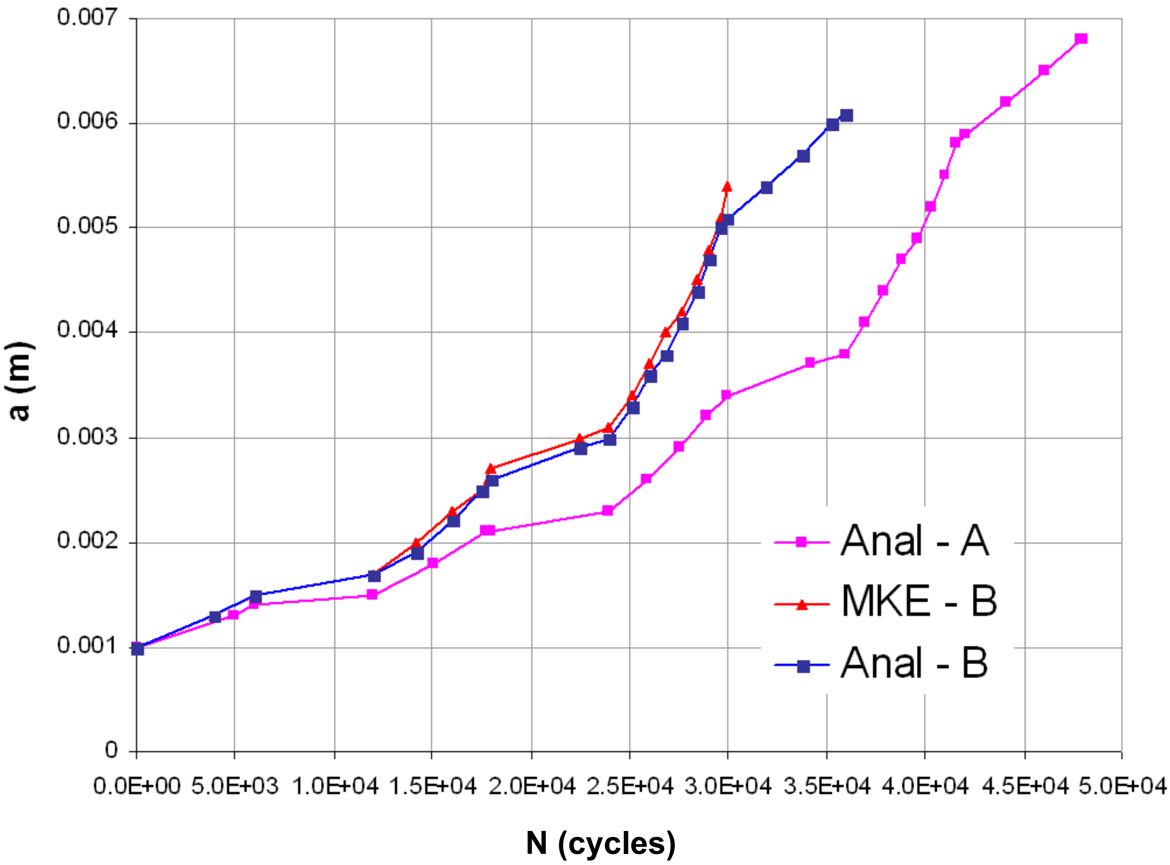

Figure 2. Results of the crack gowth analysis under load spectra

In Fig. 2 are shown relations between crack lengths a and number of cycles N during crack propagation. Curve denoted as Anal-A and Anal-B represent crack growth of points A and B of semi-elliptic surface cracks using analtic expressions (10) and (11) for SIF with one side and crack growth model (1) with other side. Curve denoted as MKE-B represents crack growth of point B where SIF is defined using approximations of FEM. In table 2 are given the complete crack growth resultats. The complete procedure for determination of SIF based on FE aproximations is illustrated in reference [8]. This procedure is based on using 3-D singular finite elements to determine SIF for several successive crack lengths and for these are established analytic expression in polynomial form for stress intensity factor (SIF) and then is used in crack growth analysis.

This method for determination of SIF's using special 3-D singular finite elements is usefull and reliable method but very complicated tool. Good agreement of crack growth results between analytic and proposed finite element approximations (MKE-B) at point B is obtained. It means that proposed analytic approach for detrrmination of SIF during crack growth is very accurate and efficient method. It is important that this analytic approach can be used instead of very complex finite element approximations to crack growth modeling at the 3-D cracked structural components.

Example 2: Numerical simulations to problems with surface cracks

Analytic expressions of SIF's for semi-elliptic surface crack at 3-D solids, defined in previous chapters, and different crack growth models are included in software package "**LOM-3**"[11]. Characteristic problem to application with respects fatigue and fracture mechanics of surface cracked problems is aircraft nose landing gear, Fig. 3. It means that numerical simulations can be used to determine SIF's using singular finite elements for different shapes of surface damages such as aircraft nose landing gear, Fig. 3, with one side and for crack growth analysis of surface cracks with other side.

Design condition with respects to „static" fracture mechanics is that SIF be lower than critical stress intensity factor or that reserve factor of safety (R.F) be larger than 1:

$$(R.F)_f = \frac{K_{IC}}{K_{I,(j=1.5)}} \qquad (12)$$

where: K_{IC} is the fracture toughness of material, K_I is the stress intensity factor, and ***j*** represents load level coefficient.

Figure 3. Structural model to nose landing gear of light training aircraft

In Fig. 4 are shown results of stress distributions to aircraft nose landing gear using finite element software MSC/NASTRAN for one load case. Using FEM here is defined critical location with respects fatigue and fracture mechanics analysis, i.e zone of maximal stresses to static part of nose landing gear, Fig. 4.

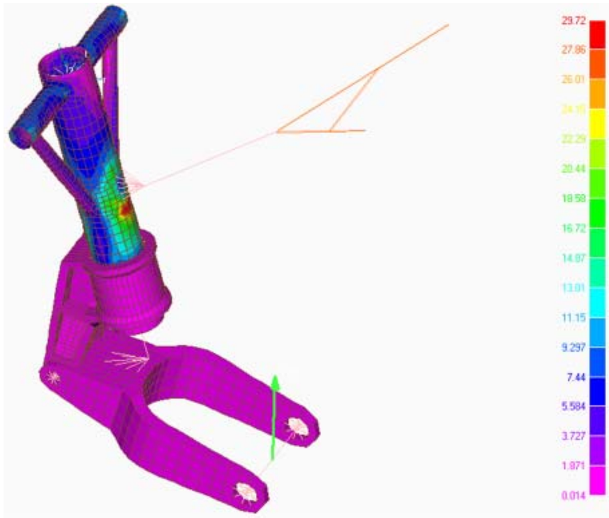

Figure 4. Finite element model of light aircraft nose landing gear

Determination of SIF using FEM to static part of nose landing gear

For „static" strength fracture mechanics analysis precise deterination of stress intensity factors is necessary. After determination of critical location to structural element using FEM with respects fracture mechanics to static part of nose landing gear, Fig. 4, it is necessary to propose initial surface crack in this location. Critical location to static part nose landing gear (steel tube), Fig 5.a. For determination of the stress intensity factors here two FE models are used: 3-D model FEM (Fig. 6) tube with semi-elliptic crack as shown in Fig. 5.b.

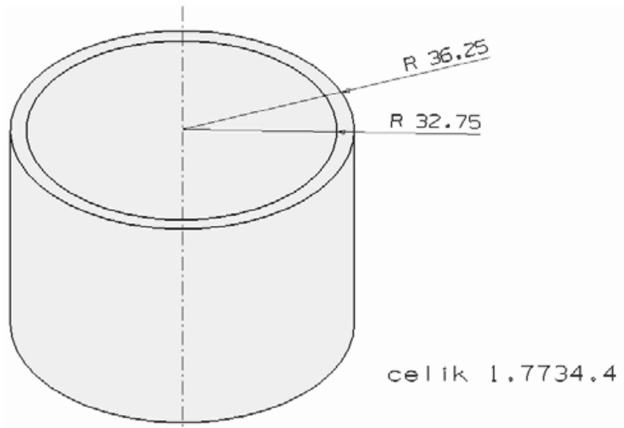

Figure 5.a. Geometry of static part under tension load (σ=10 daN/mm2)

Material: Če 1.7734.4

KIC=190 daN/mm3/2

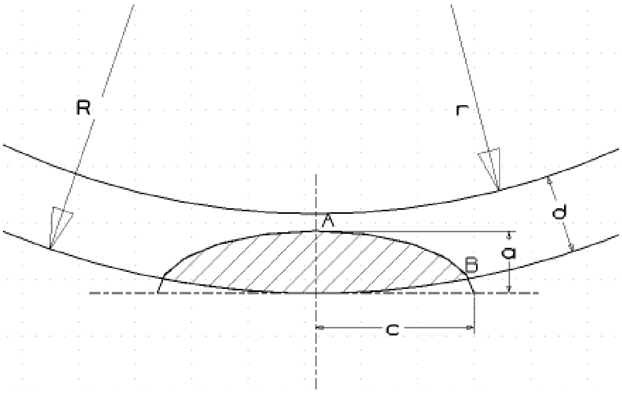

Figure 5.b. shape of semi-elliptic surface crack

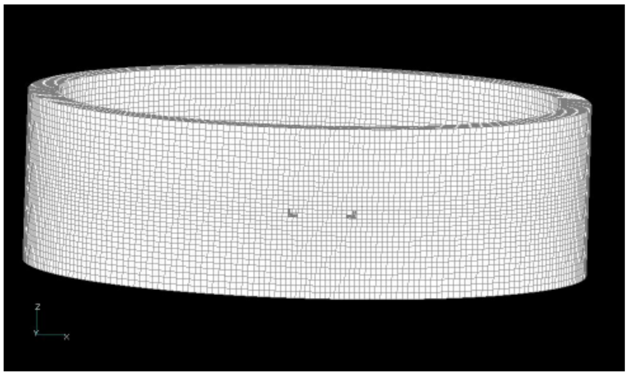

Figure 6. 3-D FEM model of static part to landing gear with semi-elliptic surface crack

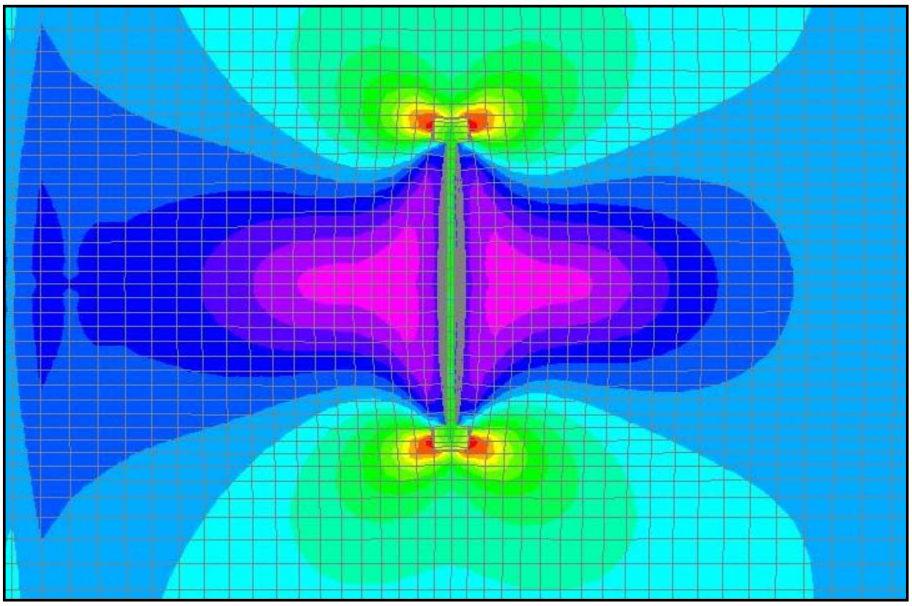

Figure 7. Detail of stress distributions around surface crack to static part of nose landing gear

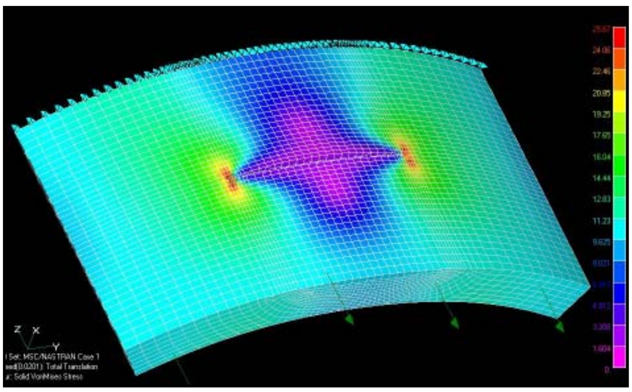

Figure 8. Detail of the stress distributions to zone semi-elliptic surface crack to static part of nose landing gear (a/c=0.4)

In figures 7 and 8 are given detail stress distributions near semi-elliptic surface crack.

Next to modeling static part using FEM with semi-elliptic surface crack, Fig. 6 to 8, carried out modeling with 2-D finite elements too, where crack is defined through the thickness of wall, as shown in Fig. 9.

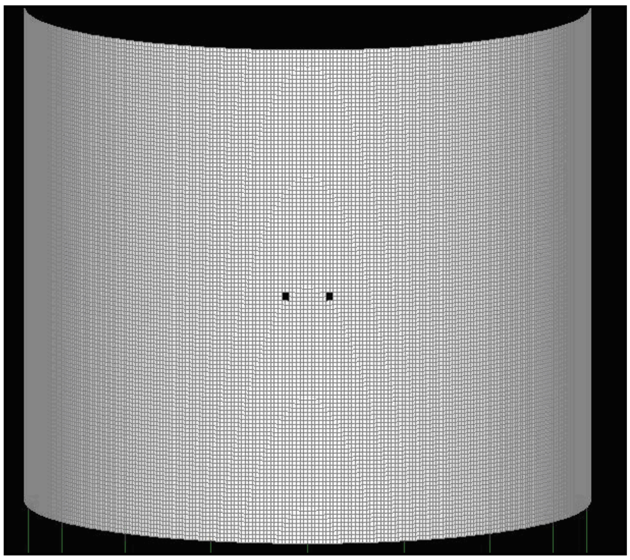

Figure 9. 2-D FEM model to static part with crack throught the thickness

Table 3. Values of SIF for cracks at static part of landing gear

	Shape of crack	KIA (daN/mm3/2)	KIB (daN/mm3/2)
c=2.75 mm	elliptic a=2.25 mm	21.28	28.56
	through the thickness	30.57	
c=7 mm	eliptična a=2.25 mm	40.25	34.5
	through the thickness	56.8	

In Table 3 are given the complete results for SIF's to surface crack, with one side, and for crack through the thickness, with other side. As expected, here are obtained larger values of stress intensity factors for through the thickness then in case of semi-elliptic surface crack.

Obtained results for SIF's, Table 3, illustrates that the last model is conservative and can be used, due its simplicity for modeling, in preliminary design [14]. First model, with surface semi-elliptical crack is real model and can be used in precise analyses to structural components with respects fracture mechanics.

CONCLUSIONS

In this investigation analytic method for determination of the stress intensity factors to surface cracks at the 3-D solid structural elements is established. Derived analytic expressions for stress intensity factors can be used for strength analyzes with respects fracture mechanics to: aircraft structural elements, fuel tanks under pressures, in ship design and many other structures. Primary attention of this paper is to establish analytic expressions of SIF's for surface crack which can be effective used in crack growth propagations and residual life estimations.. Accuracy of derived analytic expressions for SIF's are compared with finite element approximations. Good agreement analytic with finite element results is obtained in domains „static" fracture mechanics and crack growth analyses.

Practical in this investigation the efficient and accurate analytic computation procedure for crack growth analysis with initial surface cracks of robust structural components and constructions is proposed. This procedure achieves efficient approach of residual life estimation of damaged 3-D structural components under cyclic loading under cyclic loading and general load spectra. The complete computation procedure for crack growth analysis is illustrated to aircraft nose landing gears. Procedure is based on using finite element method to determine critical locations with respects to fatigue and fracture mechanics with one side and to use analytic expressions for determination of SIF's and residual life of structural components with other side.

REFERENCES

1) "Airplane Damage Tolerance Requirements", MIL-A-83444, Air Force Aeronautical Systems Division, July 1974.
2) Broek, D., Elementary Engineering Fracture Mechanics, 4th edition, Nijhoff, 1985
3) Broek, D., The Practical Uses of Fracture Mechanics, Kluwer Academic Publishers, 1989.
4) Forman, R. G., and Mettu, S. R., "Behavior of Surface and Corner Cracks Subjected to Tensile and Bending Loads in Ti-6Al-4V Alloy," Fracture Mechanics: Twenty-second Symposium, Vol. 1, ASTM STP 1131, H. A. Ernst, A. Saxena, and D. L., McDowell, eds., American Society for Testing and Materials, Philadelphia, 1992, pp. 519-546.
5) Fujimoto, W.T., and Mettu, Sambi, A, Weight Function Approach for Tracking the Growth of Surface and Radial Cracks through Residual Stress Fields or Bi-Variant Stress Fields, ASIP 2000, San Antonio, Dec. 2000.
6) Maksimović, K., Nikolić-Stanojević, V., Maksimović, S., MODELING OF THE SURFACE CRACKS AND FATIGUE LIFE ESTIMATION, ECF 16, 16th European Conference of Fracture, ECF 16, Alexandroupolis, Grčka, 2006

7) Maksimović, S., Boljanović, S., Orović, V., Komnenović, M., Fatigue Life Analysis of Damaged Structural Components Using Strain Energy Density Method, 17th European Conference on Fracture – Multilevel Approach to Fracture of Materials, Components and Structures, Brno, Czech Republic, September 3-5, 2008.

8) Maksimović, S., Fatigue Life Analysis of Aircraft Structural Components, Scientific Technical Review, Vol. LV, No. 1, 2005,pp. 29-33.

9) Maksimović, K., FATIGUE CRACK GROWTH ANALYSIS OF DAMAGED STRUCTURAL COMPONENTS UNDER MODE-I AND MIXED MODES, Scientific Technical Review, Vol. LIX Br. 1, 2009, pp. 35-40.

10) Maksimović, K., Strength analysis of constructions using damage tolerance approach under dynamic loading (In Serbian), Master Thesis, Faculty of Mechanical Engineering, University of Kragujevac, 2003.

11) Maksimović, S. i Mladenović, J., "LOM-3" Software package for fatigue strength analysis under general load spctra, VTI Žarkovo, 2001.

12) Newman, Jr., J. C. and Raju, I. S., "Prediction of Fatigue Crack-Growth Patterns and Lives in Three-Dimensional Cracked Bodies," Advances in Fracture Research (Fracture 84), Sixth International Conference on Fracture, Vol 3, 1984, pp. 1597-1608.

13) Orynyak, I.V., and Borodi, M.V., Point Weight Function Method Application for Semi-Elliptical Mode I Cracks, International Journal of Fracture, 70, No. 2, 117-124, 1994-1995..

14) Ognjanović M., Design in mechanical engineering - Multidisciplinary approach, Journal Istraživanja i projektovanja za privredu, No. 20, 2008

15) Raju, I and Neuman, J.C, Improved Stress Intensity Factors for Semi-Elliptical Surface Cracks in Finite-Thickness Plates, NASA TN X-7285, August 1977

Paper sent to revision: 06.09.2010.

Paper ready for publication: 30.11.2010.